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1989 J. Phys. A: Math. Gen. 22 L879

(http://iopscience.iop.org/0305-4470/22/18/005)

J. Phys. A: Math. Gen. 22 (1989) L879-L882. Printed in the UK

LETTER TO THE EDITOR

Correlation functions of the n-vector model with a large-scale defect in the limit $n \to \infty$

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Received 28 April 1989

Abstract. The correlation functions of the three-dimensional n-vector model are studied near the large-scale d'-dimensional defect in the limit $n \to \infty$. The model is characterised by the fact that the spin lengths close to the defect are changed with respect to their bulk value. The deviations decay as $-\lambda/r$ with the distance r from the defect. It is shown that at the critical point the correlation function exhibits non-scaling behaviour if $d' = 2$ and non-universal behaviour if $d' = 1$. The deviation of a critical exponent η is calculated up to order of λ.

The local critical behaviour of three-dimensional n-vector model near d'-dimensional defect was studied by many authors. Bray and Moore (1977) calculated the local critical exponents near the free surface. In the case of the model with an internal plane of defects, Abe (1981a, b) and Eisenriegler and Burkhardt (1982) have found that in the limit $n \to \infty$ the local exponents of the correlation functions are non-universal and vary continuously with the microscopic parameters in the defect plane. These results are in agreement with conclusions of the phenomenological approach which was developed by Bariev (1979), Fisher (1980) and Burkhardt and Eisenriegler (1981). The local non-universality of the model with a plane defect is due to the fact that the scale dimensionality of the perturbation Δ_{per} coincides with its physical dimensionality d'

$$\Delta_{\text{per}} = d' = 2. \tag{1}$$

The local non-universal behaviour in the n-vector model with the large-scale inhomogeneity on a boundary was found by Burkhardt and Guim (1982) in the limit $n \to \infty$. This model is characterised by the fact that the interaction constants close to the boundary are changed with respect to their bulk value. The deviations decay as $-\lambda/r$ with distance r from the boundary. The non-universal behaviour in this case was explained by Burkhardt (1982) and Gordery (1982).

Recently Bariev (1988), on the basis of the phenomenological approach, considered the local critical behaviour of the d-dimensional system near the inhomogeneous large-scale internal d'-dimensional defect. The value characterising the strength of the defect decay is $-\lambda/r^{\alpha}$, with r the distance from the defect. In particular this theory predicts that in the case

$$\Delta_{\text{per}} = d_{\text{ef}} \qquad d_{\text{ef}} = \max\{d - \alpha, d'\} \tag{2}$$

the correlation function on the defect plane has non-universal behaviour

$$G(R, \lambda) = R^{-2\lambda a S_{dd'}(\alpha)} G(R, 0) \tag{3}$$

where a and $S_{dd'}(\alpha)$ are constants, $G(R, 0)$ is the correlation function of the non-defect system. In the case

$$\Delta_{\text{per}} = d - \alpha = d' \tag{4}$$

the correlation function on the defect plane has a non-scaling form that manifests itself in the fact that the correlation function has a non-power dependence on distance R:

$$G(R, \lambda) = R^{-\lambda a S_d \ln R} G(R, 0). \tag{5}$$

The prediction (5) was confirmed by the exact calculations in the two-dimensional Ising model (Bariev 1989).

The aim of this letter is to test the predictions (3) and (5) on the basis of calculation in the n-vector model with an inhomogeneous large-scale defect in the limit $n \to \infty$ of the first order in λ. On the base of assumption about the validity of the exponentiation (Kadanoff and Wegner 1971, Abe 1981, Eisenriegler and Burkhardt 1982) we calculate the critical exponent of the correlation functions in the linear order in λ.

Following the work of Abe (1981) we consider the three-dimensional cubic lattice (referred to as D) on which the n-vector model is defined. The Hamiltonian of the considered model is

$$H = -\tfrac{1}{2} \sum_{j,k,m} J_{jk} \sigma_j(m) \sigma_k(m) \tag{6}$$

where the interaction constants J_{jk} depend only on $r_j - r_k$:

$$J_{jk} = \begin{cases} J & \text{if } j - k = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

For the defect-free lattice the spin length S is defined by $S_j^2 = \Sigma_m \sigma_j^2(m) = n$. Let us assume a d'-dimensional plane D' is a defect. We take coordinate axes $1, 2, \ldots, d$ in D so that axes $1, 2, \ldots, d'$ belong to D'. Abe (1981) assumed that $S_j^2 = n(1-\lambda)$ if j is in D' and $S_j^2 = n$ otherwise. In this letter we consider the model in which the defect changes the spin lengths in its vicinity. This changes the decay to $-\lambda/r$ with distance r from the defect:

$$S_j^2 = n(1 - \lambda/r_j) \qquad r_j = [j_{d'+1} + \ldots + j_3^2 + 1]^{1/2}. \tag{8}$$

It should be noted that the model under consideration is equivalent to the model in which the interaction constants between the nearest neighbours are changed. These changes are analogous to (8)

$$J_{jk} = J[(1 - \lambda/r_j)(1 - \lambda/r_k)]^{1/2} \simeq J(1 - \lambda/r_j).$$

The partition function of the n-vector model under consideration is

$$Z = \int_{-\infty}^{\infty} \prod_{k,m} d\sigma_k(m) \exp\left(\frac{1}{2} \sum_{j,k,m} K_{jk} \sigma_j(m) \sigma_k(m)\right) \prod_j \delta\left(n(1 - \lambda/r_j) - \sum_m \sigma_j^2(m)\right) \tag{9}$$

where $K_{jk} = J_{jk}/k_b T$ (k_b is the Boltzmann constant and T is the absolute temperature). In the expression (9) the δ functions may be expressed as the contour integrals

$$Z = (2\pi i)^{-N} \int_{\alpha_j - i\infty}^{\alpha_j + i\infty} \prod \mathrm{d}t_j \exp\left[n\left(\sum_j t_j(1 - \lambda/r_j) + \ln f(t_1, \ldots, t_n)\right)\right] \tag{10}$$

where

$$f(t_1, \ldots, t_n) = \int_{-\infty}^{\infty} \prod \mathrm{d}\sigma_k \exp\left(\frac{1}{2}\sum_{jk} K_{jk}\sigma_j\sigma_k - \sum_j t_j\sigma_j^2\right). \tag{11}$$

In the limit $n \to \infty$ the functional integral (10) may be calculated by the method of steepest descent. Then the saddle points $t^0, \ldots, t^0$ are determined from the equations

$$[1 - \lambda / r_j + \partial \ln f(t_1, \ldots, t_n)/\partial\, t_j]|_{t_j = t_j^0}. \tag{12}$$

Introduce a matrix A with the elements

$$A_{jk} = 2t_j^0\delta_{jk} - K_{jk}. \tag{13}$$

Then the correlation functions of the system are (Wilson and Kogut 1974, Abe 1981)

$$G(j, k) \overset{\text{def}}{=} \langle\sigma_j\sigma_k\rangle = (A^{-1})_{jk}. \tag{14}$$

Using this definition we can represent the equations (12) in the following form:

$$G(j, j) = 1 - \lambda / r_j. \tag{15}$$

The system of equations (13)-(15) is the mathematical formulation of the problem and completely defines the correlation functions of the model under consideration. For $\lambda = 0$ these correlation functions (Wilson and Kogut 1974) coincide with the correlation functions of the spherical model (Shcherbina 1988). For $\lambda \neq 0$ the procedure proposed by Abe (1981) for the n-vector model with the small-scale defect can be used. In this case the correlation function is presented as the expansion in powers of λ and the standard diagram technique can be used for the calculation of the terms of this expansion. We refer to work of Abe (1981) for more details.

As a result of these calculations we obtained to first order in λ the following expressions for the Fourier transform of the spin-spin correlation function for the case when both spins are in the defect plane:

$$G(q) = \frac{1}{2qK_c}\left\{1 - \frac{16K_c\lambda}{\pi}[\ln^2 q - \ln q \ln^2(\Lambda/2) + \ln(\Lambda/2) + \pi^2/24] + \mathrm{O}(\lambda^2)\right\} \tag{16}$$

for $d' = 2$ and

$$G(q) = -\frac{1}{2\pi K_c}[\ln q + 8K_c\lambda \ln^2(\Lambda/q) + \mathrm{O}(\lambda^2)] \tag{17}$$

for $d' = 1$. Here Λ is the parameter of cutoff at the large values of $q = [q_1^2 + \ldots + q_{d'}^2]^{1/2}$.

The results (16) and (17) correspond to the following dependences of the correlation function on distance R:

$$G(R) = G_0(R) \exp\left\{-\frac{16K_c\lambda}{\pi}[\ln^2 R + (1 + 2\ln 2 + 2\gamma)\ln R] + \mathrm{O}(\lambda^2)\right\} \tag{18}$$

(where γ is Euler's constant) for $d' = 2$ and

$$G(R) = G_0(R) R^{-[16K_c\lambda + \mathrm{O}(\lambda^2)]} \tag{19}$$

for $d' = 1$. Here $G_0(R)$ is the correlation function of the defect-free system (Wilson and Kogut 1974)

$$G_0(R) = \frac{1}{4\pi K_c}\frac{1}{R}.$$

The appearance of the term $q^{-1}\ln^2 q$ in the expansion (16) leads to the non-scaling behaviour of the correlation function. The non-scaling behaviour manifests itself in the fact that the correlation function has a non-power dependence on distance. In this situation this function does not have the critical exponent. The appearance of the term $\ln^2 q$ in the expansion (17) means the fulfilment of the necessary condition for non-universal critical behaviour near the defect plane (Kadanoff and Wegner 1971) and gives an opportunity to calculate the critical exponent η to first order in λ ($\eta = 16\lambda K_c + O(\lambda^2)$). Both these results confirm the predictions (2)-(5) of the phenomenological approach (Bariev 1988).

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